CONDITIONS OF ENCOUNTER BETWEEN DUST AND THE PLANETS

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ABSTRACT

The conditions of encounter between interplanetary dust particles and the planets are investigated for the range of particle size where the force of solar radiation is not negligible in comparison with the force of solar gravity. The action of solar radiation pressure in reducing the heliocentric orbital speed of a small dust particle makes possible very low planetocentric speeds at encounter for an entire class of heliocentric orbits of non-zero eccentricity and fairly low inclination. The speed at encounter and the apparent radiant depend on the size of the dust particle for a given heliocentric orbit throughout the range of particle size being investigated by means of dust particle sensors mounted on spacecraft.

INTRODUCTION

Dust particle sensors to be flown on spacecraft in order to directly measure the speeds and directions of motion of small dust particles having masses $m \gtrsim 10^{-12}$ gm have been developed during the past few years. These sensors were designed to be used in studying interplanetary dust particles for which the force of solar radiation is not negligible in comparison with the force of solar gravity. The range of sensitivity of the sensors extends into the regime of particle size where dust particles can be blown out of the solar system by sunlight.

The effects of solar radiation pressure must therefore be included when the measured speeds and radiants of small dust particles are used in computing the orbits. Evaluation of these effects requires investigating the conditions of encounter between a planet and interplanetary dust particles for which radiation pressure modifies the heliocentric orbital speeds. Conditions of encounter with radiation pressure included also have an important bearing on the problem of the enhanced flux of small dust particles measured in the vicinity of the earth and possibly existing also near the other planets.

THE EARTH'S DUST BELT

The flux of small dust particles measured in the vicinity of the earth through the use of rockets and satellites is higher than would be expected if radiation pressure is neglected and the orbits of small interplanetary dust particles are assumed to be similar to those of larger, meteor-producing meteoroids. This enhancement of the flux of small dust particles has been called a dust blanket by Beard (1959), a dust belt by Whipple (1961), and a dust shell by Singer (1961). The data that are presently available do not permit distinguishing among the various models envisioned by these and other investigators. A knowledge of the geocentric trajectories is critical in discussions of the nature and origin of the earth's dust belt. Referring to the phenomenon as an enhancement of the flux of small dust particles in the vicinity of the earth carries fewer connotations about the presently unknown geocentric trajectories of the dust particles and, more importantly, permits closer contact with the observational data that are now available.

The rate at which information can be obtained with a satellite-borne dust particle sensor is increased by the enhancement of the flux of small dust particles near the earth. At the same time, the complicated geocentric trajectories responsible for the enhancement of the flux make the experimental determination of the orbits of the small dust par[redacted] for meteoroids.

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The geocentric enhancement of the flux of small dust particles is shown in Figure 1, which is a cumulative flux distribution expressed in terms of the particle mass. The numerical data used in constructing Figure 1 are given in Table 1. The selected data are considered here to be representative of the available data, although Figure 1 is a corrected version of a rather outdated figure that was presented in an earlier paper. The distribution curves shown in Figure 1 come from studies of the frequency of meteorite falls, observations of meteors, analyses of results from photometric studies of the zodiacal light, and direct measurements made with dust particle sensors mounted on spacecraft.

The geocentric enhancement of the flux of small dust particles is revealed by plotting in Figure 1 the cumulative flux distributions for two different regions of space. The data from studies of the zodiacal light, meteors, and the frequency of meteorite falls are used to represent the flux of dust particles and meteoroids at earth's distance from the sun but in regions removed from the earth. The direct measurements made in the vicinity of the earth are used to represent the flux of small dust particles near the earth. The flux of meteoroids is essentially the same for both regions of space, since the known distributions of orbits and the high speeds of encounter (20 to 40 km/sec, on the average, as determined by radar and photographic studies of meteors) preclude any appreciable geocentric enhancement of the flux of meteoroids. The flux of 10^{-8} particles/m^2/sec/2π ster for meteoroids having masses $m \sim 1.3 \times 10^{-6}$ gm observed by Elford, et al. (1964) through the use of radar indicates that no measurable geocentric enhancement of the flux occurs for the faint radar meteors.

Comparison of the flux of small dust particles in interplanetary space with the higher flux measured in the vicinity of the earth leads to the conclusion that a geocentric enhancement of the flux of small dust particles exists. The degree of enhancement of the flux and the particle mass at which the flux

begins to be enhanced both depend on which of the various size distributions derived from studies of the zodiacal light and solar F-corona is used to represent the flux of small dust particles in interplanetary space. The size distribution derived by Ingham (1961) and shown in Figure 1 represents the best fit to the distribution of surface brightness of the zodiacal light that was observed by Blackwell and Ingham. This size distribution, if extended to smaller sizes of dust particles, is in good agreement with the size distribution of small dielectric spheres (water ice) used by Giese (1963) in calculating models for the zodiacal light. The photoelectric observations of the surface brightness and polarization of the zodiacal light reported by Weinberg (1964) are fitted more closely by the size distribution given by Giese than by any of the other size distributions that have been suggested. The size distribution given by Elsässer (1954) has subsequently been altered and was included in Figure 1 only to indicate the desirability of a smooth transition between the cumulative flux distribution for the zodiacal dust particles and the one for the radar meteors.

The geocentric enhancement of the flux of small dust particles, as portrayed in Figure 1, begins to appear at a particle mass between 10^{-6} and 10^{-7} gm and reaches a maximum ($\sim 10^4$ enhancement) for dust particles having masses in the neighborhood of $\dot{m} \sim 10^{-11}$ gm. This dependence suggests that some non-gravitational force, such as that of solar radiation pressure, may play an important role in producing the geocentric enhancement of the flux of small dust particles. If so, similar enhancements of flux can be expected to exist in the vicinity of other planets. Investigation of this possibility again requires that radiation pressure be includedin the conditions of encounter between small interplanetary dust particles and the planets.

The following sections are devoted to an approximate treatment of the conditions of encounter between the planets and

small interplanetary dust particles. Solar radiation pressure is included in the treatment, but other non-gravitational forces such as cospuscular drag, Coulomb drag, and Lorentz forces are neglected in this first order approximation. Any force which effectively reduces the central gravitational force exerted by the sun can produce effects on the motion of a small dust particle similar to the effect of radiation pressure.

EQUATIONS OF MOTION AND THE POYNTING-ROBERTSON EFFECT

Some of the equations to be used here are sufficiently well-known that they really need not be presented, but they do provide a convenient means for introducing parameters needed in the discussion. The scalar equations of motion given by Robertson (1937) or by Wyatt and Whipple (1950) form a convenient starting point.

Expressed in polar coordinates (r, θ), the equations for the heliocentric motion of a dust particle of mass m moving in the combined gravitational and radiation fields of the sun of mass $M_\odot \gg m$ are

$$\frac{d^2r}{dt^2} - r\left(\frac{d\theta}{dt}\right)^2 = -\frac{\mu_{eff}}{r^2} - \frac{2\alpha}{r^2}\frac{dr}{dt} \qquad \text{(radial)}$$

and

$$\frac{1}{r}\frac{d}{dt}\left(r^2\frac{d\theta}{dt}\right) = -\frac{\alpha}{r}\frac{d\theta}{dt} \qquad \text{(angular)}$$

where

$$\mu_{eff} = \mu - \alpha c, \quad \mu = GM_\odot, \text{ and } \alpha = \frac{3L_\odot}{16\pi c^2}\frac{Q_{pr}}{\delta s} .$$

In the foregoing, G is the constant of gravity, $L_\odot$ is the luminosity of the sun, c is the speed of light, δ is the mass density of the dust particle, and s is the particle radius (all measured in cgs units). The parameter Q_{pr} is the radiation pressure efficiency factor defined by

$$C_{pr} = Q_{pr}\pi s^2$$

where C_{pr} is the cross-sectional area of the dust particle effective in intercepting sunlight to produce radiation pressure

on the dust particle.

The magnitude of the force of radiation is

$$F_r = u(r)\ C_{pr}$$

where $u(r)$ is the energy density of sunlight at a heliocentric distance r. The magnitude of the force of gravity is

$$F_g = \frac{\mu m}{r^2} .$$

A dimensionless parameter β defined as

$$\beta = \frac{F_r}{F_g} = \frac{3L_\odot}{16\pi c\mu} \frac{Q_{pr}}{\delta s}$$

will be used extensively throughout the following discussions. The parameter β is simply the magnitude of the force of radiation measured in units of the force of gravity. Both the flux of sunlight and the force of solar gravity vary inversely as the square of the distance from the sun, so the parameter β is independent of the heliocentric distance. Other non-gravitational forces could be introduced at this point by using an additional parameter β' which would not (in general) be independent of the heliocentric distance. The expression given earlier for β becomes, upon substitution of the appropriate numerical values for the various constants.

$$\beta = 5.78 \times 10^{-5} \frac{1}{\delta s} \frac{gm}{cm^2} .$$

The equations of motion may be written also in terms of the dimensionless parameter β as

$$\frac{d^2r}{dt^2} - r\left(\frac{d\theta}{dt}\right)^2 = \frac{-\mu}{r^2} + \frac{\mu\beta}{r^2}\left(1 - \frac{1}{c}\frac{dr}{dt}\right) - \frac{\mu\beta}{r^2}\ \frac{1}{c}\ \frac{dr}{dt}$$

and

$$\frac{1}{r}\ \frac{d}{dt}\left(r^2\frac{d\theta}{dt}\right) = -\frac{\mu\beta}{r^2}\ \frac{r}{c}\frac{d\theta}{dt} ,$$

since β and α are simply related by the expression

$$\beta = \frac{\alpha c}{\mu} .$$

Radiation pressure produces several effects on the motion of a small dust particle. A dust particle moving in a heliocentric orbit receives sunlight from a source that is moving relative to the dust particle, so the incident sunlight undergoes aberration through a small angle

$$\varphi \sim \frac{r}{c}\frac{d\theta}{dt}$$

which can be neglected in a first approximation. The component of motion of the dust particle along the position vector of the dust particle causes sunlight of frequency ν to be shifted through the Doppler effect to a new frequency ν' given by

$$\nu' = \nu\left(1 - \frac{1}{c}\frac{dr}{dt}\right).$$

The radial motion of the dust particle and the shift in frequency of the incident sunlight produce a corresponding change in the energy density of the intercepted radiation and, hence, the radiation pressure exerted on the dust particle. The terms

$$\frac{\mu\beta}{r^2}\ \frac{1}{c}\ \frac{dr}{dt} \quad \text{and} \quad \frac{\mu\beta}{r^2}\ \frac{r}{c}\ \frac{d\theta}{dt}$$

represent the radial and tangential components, respectively, of the acceleration produced by this drag force. The equations of motion given previously are correct to the first order in the ratio of the speed of the dust particle to the speed of light. The problem was first formulated correctly by Robertson (1937) who used a relativistic treatment in order to derive the generalized equations of motion. The action of sunlight in effectively reducing the central gravitational force of attraction by the sun and in giving rise to the non-conservative drag force is called the Poynting-Robertson effect.

The energy and angular momentum no longer constitute constants of motion because of the presence of the non-conservative drag force introduced by radiation pressure. Integration of the angular equation of motion yields

$$h = r^2 \frac{d\theta}{dt} = ho - \frac{\mu\beta\theta}{c}$$

where h is the angular momentum (per unit mass) of the dust particle and h_o is the value of h at some epoch, $t = t_o$. As time t (or θ) increases, the angular momentum decreases. The dust particle loses angular momentum and energy of motion, causing it to spiral toward the sun. The dynamical evolution of the orbits of meteoroids under the Poynting-Robertson effect has been considered by Wyatt and Whipple (1950) for elliptic orbits as well as for the circular orbits considered by Robertson (1937).

Temporarily dropping the small terms in

$$\frac{1}{c}\frac{dr}{dt} \text{ and } \frac{r}{c}\frac{d\theta}{dt}$$

from the equations of motion leaves

$$\frac{d^2r}{dt^2} - r\left(\frac{d\theta}{dt}\right)^2 \doteq -\frac{\mu}{r^2}(1-\beta)$$

and

$$\frac{1}{r}\frac{d}{dt}\left(r^2\frac{d\theta}{dt}\right) \doteq 0.$$

The radial equation of motion now displays only the action of solar radiation pressure in effectively reducing the central force of gravity. A value $\beta > 1$ makes the right-hand side of the radial equation of motion positive and causes the central force to become one of repulsion. A dust particle for which $\beta > 1$ will be blown out of the solar system by radiation pressure, while one for which $\beta = 1$ will not be subject to any acceleration by the sun.

Integration of these approximate equations of motion yields the orbital speed v of the dust particle as

$$v^2 = \mu(1-\beta)\left(\frac{2}{r} - \frac{1}{a}\right)$$

where a is the semi-major axis of the orbit of the dust particle.

Comparison with the analogous equation

$$v^2 = \mu(\frac{2}{r} - \frac{1}{a})$$

for a dust particle which is sufficiently large that $\beta \doteq 0$ shows that radiation pressure reduces the orbital speed of a small dust particle below that for the purely gravitational case. A small dust particle moving in a planet-like orbit moves more slowly than the planet and therefore has a non-zero speed relative to the planet. This effect was noted by Poynting in 1912, soon after the existence of radiation pressure had been experimentally verified.

An assumption commonly used in simplified treatments of this type is that a dust particle is perfectly absorbing and absorbs sunlight over an area equal to the geometric cross-sectional area of the particle. Values of β computed under this assumption ($Q_{pr} = 1$) are shown graphically in Figure 2 for various values of the mass density δ and particle radius s. As was noted previously, a value $\beta > 1$ means that the dust particle will be blown out of the solar system by radiation pressure. The values $\beta = 1$ occur in Figure 2 for particle radii $s \sim 1\mu$ which are comparable to the wavelength of sunlight ($\sim 0.55\mu$). The assumption that classical optics could be used to compute β fails. The radiation pressure efficiency factor Q_{pr} (and, hence, β) is one of the parameters which can be computed through use of the Mie theory of light-scattering by spherical particles of arbitrary size and known index of refraction. It is unlikely that interplanetary dust particles are isotropic homogeneous spheres, but one presently has little choice in computing the values of β other than to assume a composition for which the index of refraction is known and to use the Mie theory for computing Q_{pr}.

Further treatment of the rather complicated relations among particle mass or size, mass density, composition, structure, shape, and the parameter β are beyond the scope of this work. An extensive treatment of the Mie theory can be found in the book by van de Hulst (1957). The important points in the present

discussion can be adequately illustrated by taking $Q_{pr} = 1$ and using Figure 2 for relating β to the physical parameters of the dust particles. A scale of particle mass computed for a mass density of $\delta = 2.5$ gm/cm^3 has been added along the top border in Figure 2 for this purpose.

The direct measurements apply for dust particles having masses $m \lesssim 10^{-7}$ gm. Reference to Figure 2 shows that such dust particles have values of $\beta \gtrsim 0.1$, approximately, for any of the values of mass density δ that are represented. A value $\beta = 0.01$ might be considered to be negligible in comparison with unity, but such a value applies only for the upper end of the range of particle mass encompassed by the direct measurements. The majority of the direct measurements apply for dust particles having masses between about 10^{-11} gm and 10^{-8} gm. A dust particle having a mass $m = 10^{-11}$ gm and a mass density $\delta = 2.5$ gm/cm^3 has a radius of about 1 μ and, from Figure 2, a value of $\beta \doteq 0.23$, which is hardly negligible compared to unity.

The dependence of the speed at encounter (entry of the dust particle into the sphere of influence of a planet) on the value of β as the mass of the dust particle is varied from $m \sim 10^{-7}$ gm down to any limits ($\beta = 1$) set by radiation pressure is to be emphasized. The treatment is approximate because of the simplifying assumptions made in relating β to the particle mass or the particle size. It is also approximate to the extent that the heliocentric motion of a dust particle is followed by using the two-body equations of motion presented earlier with radiation pressure included. As the dust particle enters the planet's sphere of influence, gravitational control is transferred to the planet, and the motion of the dust particle in the region of the planet is treated as a two-body problem. The small tangential drag force produced by radiation pressure can be neglected during an encounter but should be included in the motion of a dust particle between encounters.

The orbital speed of a dust particle at encounter is given by

$$v^2 = \mu(1-\beta)\ (2 - \frac{1}{A})$$

in which the semi-major axis a of the heliocentric orbit of the dust particle is measured in units of the semi-major axis a_o of the planet's orbit (assumed to be circular) as

$$A = \frac{a}{a_o}.$$

The notation to be used here is that employed by Opik (1951). The relative speed can then be found by vector addition of the velocity of the dust particle and the velocity of the planet. Alternatively, and much more simply, the problem can be formulated as a Jacobi three-body problem with radiation pressure included. The speed U of the dust particle at encounter (measured relative to the planet and expressed in units of the circular speed v_c of the planet) is given by

$$U^2 = (\frac{v}{v_c})^2 = 1 - 2\sqrt{1-\beta}\sqrt{A(1-e^2)}\cos i + (1-\beta)(2 - \frac{1}{A})$$

in which e is the eccentricity of the heliocentric orbit of the dust particle and i is the inclination of the heliocentric orbit of the dust particle measured relative to the plane of the planet's orbit. Setting $i = 0^o$ and replacing A by Q where

$$Q = A(1-e),\ q = a(1-e),\ \text{and}\ Q = \frac{q}{a_o}$$

yields

$$U^2 = 1 - 2\sqrt{1-\beta}\sqrt{Q(1+e)} + (1-\beta)\ (2 - \frac{(1-e)}{Q}).$$

The foregoing expression reduces to

$$U^2 = 1 - 2\sqrt{1-\beta}\sqrt{1+e} + (1-\beta)\ (1+e)$$

for encounter at the perihelion passage (Q = 1) of the dust particle. This expression is quadratic and has the root

$$U = 1 - \sqrt{1-\beta}\sqrt{1+e}$$

Now, $U = 0$ for $\beta = \beta_o$ where β_o and e are related by the expression

$$\beta_o = 1 - \frac{1}{1+e} .$$

The ranges of β_o and e for which $U = 0$ can occur when radiation pressure is included are

$0 \leq \beta_o < 0.5$, $0 \leq e < 1$, (elliptic orbits)

$\beta_o = 0.5$, $e = 1$, (parabolic orbits)

$0.5 < \beta_o < 1$, $e > 1$, (hyperbolic orbits).

Circular heliocentric orbits ($e = 0$) give $U = 0$ only for $\beta = 0$, which applies to dust particles considerably larger than those studied through the direct measurements technique. Figure 3 shows U (or v/v_c) as a function of β for circular orbits and for several values of the inclination. The ordinate on the right will be explained and used later. It should be noted that small dust particles having $\beta \gtrsim 0.01$ and planet-like orbits do not have particularly low speeds of encounter (even for zero inclination) when the effect of radiation pressure on the orbital speed of a dust particle is included.

Figures 4a and 4b show U (or v/v_c) as a function of β for two values of eccentricity e arbitrarily chosen such that β_o falls in the range of particle size ($\beta \gtrsim 0.01$) encompassed by the direct measurements. Figures 3, 4a, and 4b show that very low speeds of encounter between planets and small dust particles can occur for dust particles having non-circular heliocentric orbits of fairly low inclination. Further calculations show that the singular solutions shown in Figures 4a and 4b are but two of an entire class of orbits for which small dust particles can have low speeds at encounter. Figure 5 shows the (A^{-1}, e) domain in which the clear areas contain values of A and e for which planetary encounters are possible. Representative values of β_o are noted along the line for perihelion intercepts. The class of singular solutions which give $U = 0$ for $i = 0^o$ forms a line singularity. An eccentricity $e \gtrsim 0.01$ is required in order for β_o to fall in the range ($\beta \geq 0.01$, $m \lesssim 10^{-7}$ gm)

encompassed by the direct measurements. The entire class of orbits for which $i \lesssim 5^o$, $e \gtrsim 0.01$ provides small dust particles which have low speeds at encounter provided the encounter occurs near the perihelion passage of a dust particle. The action of radiation pressure in reducing the perihelion distance of a dust particle through the Poynting-Robertson effect further serves to continuously move orbits of low inclination into position for the low speed encounters to occur. The line singularity appearing at $Q = 1$ in the (A^{-1}, e) domain for $i = 0^o$ presents no problem regarding an "infinite" enhancement of the flux in the vicinity of a planet, because the number of dust particles having such orbits is finite. The orbits of primary interest in the problem of the enhanced flux near a planet are those in the neighborhood of the line $Q = 1$, $0.1 \lesssim e < 1$, $i = 0^o$, in the (A^{-1}, e) domain.

The component of U normal to the plane of the planetary orbit is given by

$$U_z = \pm \sqrt{1-\beta}\ \sqrt{A(1-e^2)}\ \sin i,$$

so increasing i from 0^o adds a non-vanishing component to U. fairly rapidly through the dependence of U_z on sin i. The factor $\sqrt{1-\beta}$ broadens the range of i over which a given small value of U_z can be obtained. The component of U along the heliocentric position vector of a dust particle is

$$U_x = \pm \sqrt{1-\beta} \left\{ (2 - \frac{1}{A}) - A(1-e^2) \right\}^{1/2}.$$

A non-vanishing component is added to U fairly rapidly as Q is reduced below 1. Again, the factor $\sqrt{1 - \beta}$ reduces the magnitude of the non-vanishing component. Evaluation of the degree of enhancement of the flux of small dust particles which will occur near a planet therefore requires an integration over the orbital elements a, e, and i as well as over a size distribution or a distribution of the values of β for the dust particles encountered by the planet. The writer has not yet been successful in performing this integration.

The action of radiation pressure in reducing the orbital speed of a small dust particle also affects the apparent radiant of the dust particle. Varying the size of a dust particle so that the value of β crosses a singularity of the type shown for $i = 0^{o}$ in Figures 4a and 4b causes the radiant of the dust particle to shift by 180^{o}. Dust particles having orbits with $e > 0$, $i \doteq 0^{o}$, and $\beta < \beta_o$ appear to overtake the planet, while those in similar orbits but having $\beta > \beta_o$ appear to be overtaken by the planet. If a planet encounters a cloud of dust particles all having similar orbits of low inclination and a distribution of values of β encompassing the value of β_o appropriate for the eccentricity of the orbits, the dust particles can appear to enter the planet's sphere of influence from all directions. Such a cloud of dust particles would, of course, be in the process of dispersing because of the differential effects of radiation pressure on the heliocentric orbits of the dust particles.

The effects of radiation pressure on the conditions of encounter that were described in the preceding paragraphs apply also to the small dust particles detected by a dust particle sensor mounted on an interplanetary probe such as Mariner IV. The effects of radiation pressure on the speeds and radiants of the dust particles detected by such an instrument need to be investigated in greater detail.

The effect of radiation pressure on the accretion process has some implications (the importance of which has not yet been investigated) regarding the formation of planetesimals in a rotating circumstellar nebula out of which a star forms. As a newly-formed star becomes luminous and clears away the gas in the nebula, the small dust particles which are not blown away by radiation pressure assume Keplerian orbits about the star. The accretion process may then change because of the differential effects of radiation pressure on the planetesimals

and the smaller dust particles being accreted by the planetesimals. The solar system is in a stage somewhat analogous to the latter. The nebular gas is gone, so the planets are now accreting small dust particles which have their heliocentric motion modified by radiation pressure and the radial flow of the solar wind.

GRAVITATIONAL ENHANCEMENT OF THE FLUX OF INTERPLANETARY DUST PARTICLES

The geocentric enhancement of the flux of small dust particles has been the subject of considerable discussion both before and after its existence was recognized in the results obtained with dust particle sensors flown on rockets and satellites. Whipple (1961) suggested that the dust particles were the product of meteoroidal impacts on the moon. The majority of the remaining proposed explanations for the existence of the enhanced flux basically depend on the gravitational concentration of interplanetary dust particles streaming at low speed into the vicinity of the earth. No attempt is made here to review the numerous variations on this theme, because the effects of radiation pressure on the conditions of encounter have been neglected.

The gravitational enhancement of the flux of interplanetary dust particles entering the vicinity of the earth has been treated through the use of Liouville's theorem by Singer (1956, 1961) and by Shelton, et al. (1964). The three-body problem (consisting of the sun, the earth, and a dust particle) was treated as a succession of two-body approximations. Singer (1961) assumed that the velocities of the dust particles were isotropic at infinity (entry into the earth's gravitational field).

The degree of enhancement of the flux depends on the average speed at which dust particles enter the vicinity of a planet. The requirements on the speed of encounter can be illustrated by a very brief treatment using Liouville's theorem and following closely the expositions given by Singer (1961) and by Shelton, et al. (1964). The general treatment applies for any planet, with the degree of enhancement of the flux depending rather

sensitivity on the distributions of a, i, and β for the dust particles encountered by the planet. The remainder of the present discussion is limited, however, to the problem of the geocentric enhancement of the flux of dust particles.

The densities at two points $(\vec{r}_g, \vec{v}_g)$ and $(\vec{r}'_g, \vec{v}'_g)$ on a dynamic trajectory in a six-dimensional phase space composed of the geocentric position and velocity vectors $\vec{r}_g$ and $\vec{v}_g$ are related through Liouville's theorem by

$$n\,(\vec{r}_g, \vec{v}_g) = n\,(\vec{r}'_g, \vec{v}'_g)$$

if no particles are lost. The density of dust particles in real space at a geocentric distance r_g is obtained by integrating $n(\vec{r}_g, \vec{v}_g)$ over velocity space to get

$$\rho(r_g) = \int_{u_g} n(\vec{r}_g, \vec{v}_g)d\vec{v}_g$$

The scalar flux of dust particles at a geocentric distance r_g is

$$\Phi(r_g) = \rho(r_g)\ v_g(r_g)$$

taking r'_g, v'_g, ρ_∞, and Φ_∞ to apply when the dust particles enter the earth's gravitational field leads to

$$\rho\frac{(r_g)}{\rho_\infty} = f\ \left[1 + \frac{v_g^2(r_g)}{v_\infty^2}\right]^{1/2}$$

and

$$\Phi\frac{(r_g)}{\Phi_\infty} = f\ \left[1 + \frac{v_g^2(r_g)}{v_\infty^2}\right]$$

as the density enhancement and flux enhancement, respectively, at a geocentric distance r_g. The factor f allows for those dust particles which are absent from the receding flux because they collided with the earth. The algebraic expression for the factor f is given by Shelton, et al. (1954). The numerical values of f range between 0.5 and 1. The speed at infinity v_∞ corresponds to the speed at encounter v used earlier in the discussion of the conditions of encounter.

The conservation of energy of geocentric motion yields

$$v_g^2(r_g) = v_\infty^2 + v_e^2(r_g)$$

where

$$v_e(r_g) = \sqrt{\frac{2\mu_\oplus}{r_g}}$$

is the speed of escape from the earth at a distance r_g. At the altitudes of the near-earth satellites ($r_g \sim 7000$ km), the speed of escape is

$$v_e \sim 10 \text{ km/sec.}$$

For speeds at encounter

$$v_\infty << v_e \sim 10 \text{ km/sec,}$$

one has

$$v_g^2 (r_g) \sim v_e^2 (r_g)$$

so that the density and flux enhancements reduce to

$$\frac{\rho(r_g)}{\rho_\infty} \sim f \frac{v_e(r_g)}{v_\infty}$$

and

$$\frac{\Phi(r_g)}{\Phi_\infty} \sim f \frac{v_e^2(r_g)}{v_\infty^2} .$$

An enhancement of the flux by a factor $\sim 10^4$ therefore requires $v_\infty \sim 0.1$ km/sec. The concept of the sphere of influence inherent in the simplified treatment using Liouville's theorem encounters trouble, for 0.1 km/sec is not small compared to the speed of escape at the distance of the boundary of the sphere of influence for the earth. A realistic treatment of the problem therefore requires numerically integrating the three-body equations of motion with radiation pressure included.

The orbits for which very low speeds at encounter are possible for dust particles of various sizes were discussed earlier and represented in Figures 3, 4a, 4b, and 5. The speed at encounter v measured in units of the speed of escape v_e at the altitudes of the near-earth satellites is given as the ordinate on the right in Figures 3, 4a, and 4b.

Singer (1961) assumed that relatively large dust particles ($s \sim 200\mu$) have almost circular heliocentric orbits of very low inclination ($e \doteq 0, i \doteq 0^o$) in order to justify taking $v_\infty = 1$ km/sec, which leads to a flux enhancement $\sim 10^2$. A subclass of these orbits was investigated by Dole (1962), who numerically integrated the geocentric trajectories of dust particles having initially circular heliocentric orbits of zero inclination ($e = 0$, $i = 0^o$). Southworth (1963) has criticized the two-dimensional model used by Dole and has argued that the fraction of zodiacal dust particles having such heliocentric orbits must be quite small.

A dust particle having a radius $s \sim 200\mu$ and a mass density of 1 gm/cm^3 has a value of $\beta \sim 2.4 \times 10^{-3}$, which justifies the neglect of radiation pressure by Singer (1961). But the same dust particle has a mass $m \sim 3.3 \times 10^{-5}$ gm, which removes it from the regime of particle mass covered by the direct measurements and places it in the regime of radar meteors. The available direct measurements apply for dust particles having masses $m \lesssim 10^{-7}$ gm, with the maximum enhancement of the flux occurring for dust particles having masses in the neighborhood of $m \sim 10^{-11}$ gm. A dust particle having a mass $m = 10^{-11}$ gm and a mass density $\delta = 2.5$ gm/cm^3 has a radius $s \doteq 1\ \mu$. The value of β is about 0.23, according to Figure 2. The speed of encounter for such a dust particle having an earth-like orbit is about 3.6 km/sec. Taking this as an average speed yields a flux enhancement of only 8, which is very much less than the factor $\sim 10^4$ given by the direct measurements.

The shortcomings of the treatments by Singer (1961) and Dole (1962) are shared by numerous other proposed explanations for the geocentric enhancement of the flux of small dust particles in which radiation pressure was neglected. The work of Dole was mentioned here because such numerical integrations should be extended to a three-dimensional model and to the case in which radiation pressure is not neglected. The formulation of the problem of the gravitational enhancement of flux in terms Liouville's theorem that was given by Singer (1961) is useful, provided two

important changes are made. These are (1) radiation pressure should be included so that the discussion can be directed to the range of particle size for which the measured geocentric enhancement of the flux occurs, and (2) the assumption that the small dust particles which produce the zodiacal light have predominantly circular heliocentric orbits of very low inclination should be avoided. The assumption that small dust particles having orbits of fairly low inclination contribute most to the enhancement of the flux near a planet cannot be avoided, but the restriction on the range of the inclination can be relaxed slightly because of the presence of the term $\sqrt{1 - \beta}$ in the expression for U_z. The low speed encounters occur only for non-circular orbits of fairly low inclination when dust particles of the sizes involved in the measured enhancement of the flux are considered.

The remaining unsolved problem consists of integrating over the unknown distribution of orbits and the rather poorly determined size distribution for the small interplanetary dust particles in an attempt to ascertain whether appreciable planetocentric enhancements of the flux can occur directly. The boundary conditions which must be met by any distributions of orbits and size distributions assumed for use in such an integration include the observed distribution of surface brightness and polarization of the zodiacal light over the celestial sphere.

SUMMARY

The conditions of encounter between planets and small interplanetary dust particles have been investigated in an approximate manner. Solar radiation pressure has been included in the conditions of encounter, and the dependence of the speed at encounter on the size of a dust particle has been emphasized.

The low speeds of encounter which occur near perihelion passage encompass an entire class of non-circular heliocentric orbits of fairly low inclination when dust particles of the sizes being studied through the use of spacecraft are considered. This

class of orbits provides dust particles which can encounter the planets at the very low speeds necessary in order for a gravitational enhancement of the flux to occur. Evaluation of the effectiveness of this class of orbits in directly producing the enhanced flux of small dust particles measured in the vicinity of the earth is made difficult by the present lack of knowledge about the orbits of small dust particles both in the vicinity of the earth and in interplanetary space. The source of the small dust particles and the mechanism for the enhancement of the flux measured in the vicinity of the earth through the use of spacecraft still remain to be satisfactorily explained.

CAPTIONS FOR FIGURES

Figure 1. Cumulative flux distribution compiled from the data in Table I to show the enhancement of the flux of small dust particles in the vicinity of the earth.

Figure 2. Dependence of the ratio (β) of the force of radiation to the force of gravity on the size and mass density of a dust particle.

Figure 3. Dependence on particle size (or β) of the speed at encounter for dust particles and a planet when both are moving in circular heliocentric orbits.

Figure 4a, 4b. Two examples of the dependence on particle size (or β) of the speed at encounter for a planet moving in a circular orbit and dust particles moving in elliptic heliocentric orbits.

Figure 5. The (A^{-1}, e) domain showing the orbits for which planetary encounters are possible and the line singularity next to which are clustered the orbits that yield very low speeds of encounter for small dust particles in orbits of low inclination.

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TABLE 1

Selected Observational Data on the Mass Distribution of Interplanetary Dust Particles

Type of Observation	Source	Equation of Distribution	Range of Validity
Meteor Observations:			
	Watson (1941, 1956)	$\log N(M_v) = 5.87 + 0.4\,M_v$	$-3 \le M_v \le 10$
	Millman & Burland (1956) [by McKinley (1961)]	$\log N(M_v) = 6.3 + 0.4\,M_v$	$3 \le M_v \le 10$
		$\log N(M_v) = 6.0 + 0.50\,M_v$	$0 \le M_v \le 3$
		$\log N(M_v) = 6.0 + 0.57\,M_v$	$-10 \le M_v \le 0$
	Hawkins & Upton (1958)	$\log N(M_v) = 4.93 + 0.538\,M_v$	$0 \le M_v \le 4.1$
	Hawkins (1959)	$\log N(M_v) = 4.52 + 0.4\,M_v$	$-10 \le M_v \le -3$
	Kaiser (1961)	$\log N(M_r) = 4.86 + 0.468\,M_r$	$8 \lesssim M_r \lesssim 10.8$
Meteorite Collections:	Brown (1960, 1961)	$\log N(m) = 3.5 - 0.8 \log m$	$10^4 \lesssim m \lesssim 10^{11}$ gm
Zodiacal Light Studies:			
	Elsässer (1954)	$\log I(s) = -14.32 - 2.5 \log s$	$2\mu \le s \le 0.2$ cm
	Ingham (1961)	$\log I(s) = -16.38 - 3.0 \log s$	$0.4\mu \le s$
Direct Measurements:	McCracken, et al. (1961)	$\log I(m) = -17.30 - 1.70 \log m$	$10^{-10} \lesssim m \lesssim 10^{-6}$ gm

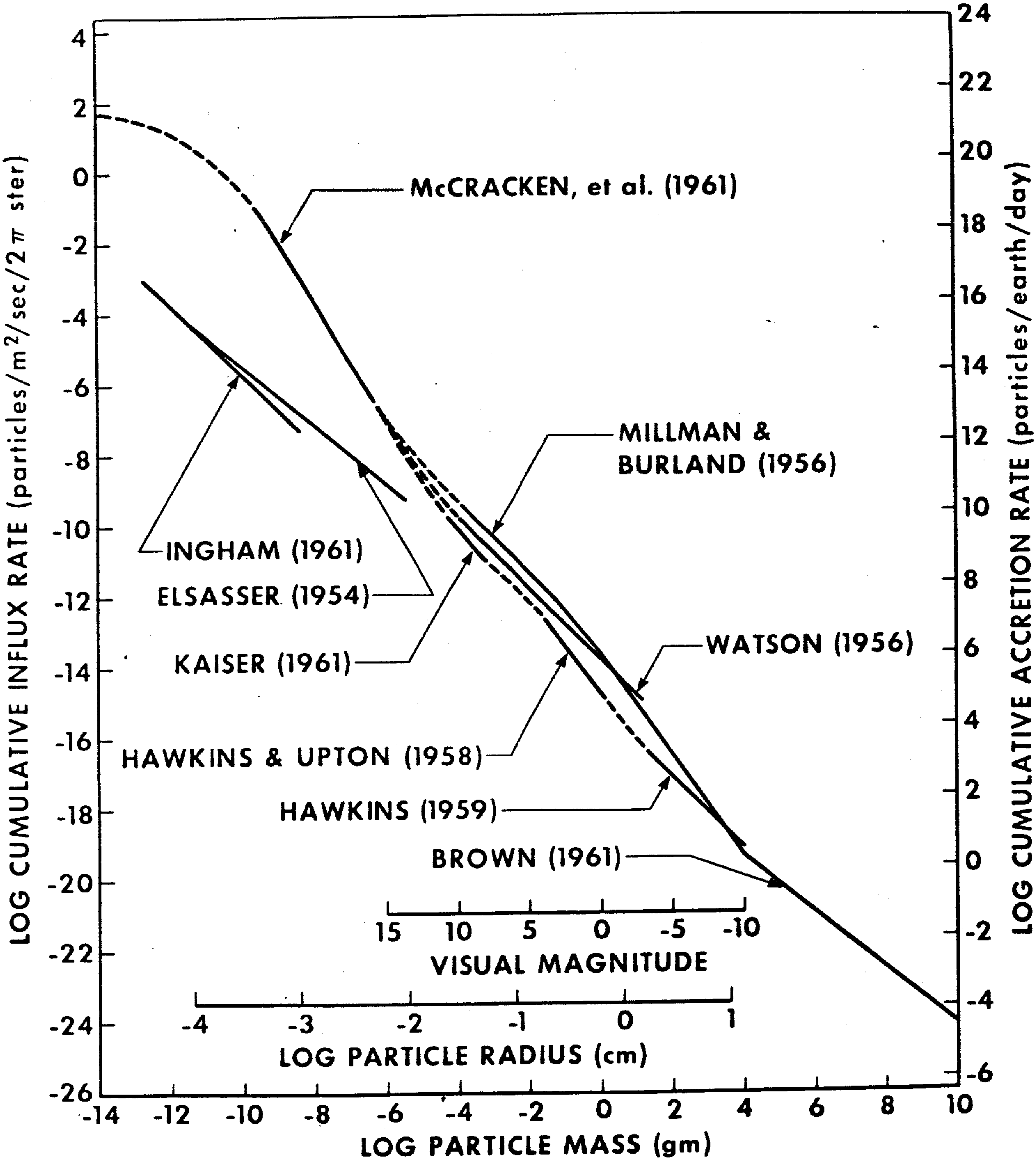
N( ) = cumulative accretion rate in particles/earth/day

I( ) = cumulative flux in particles/m^2/sec/2π ster

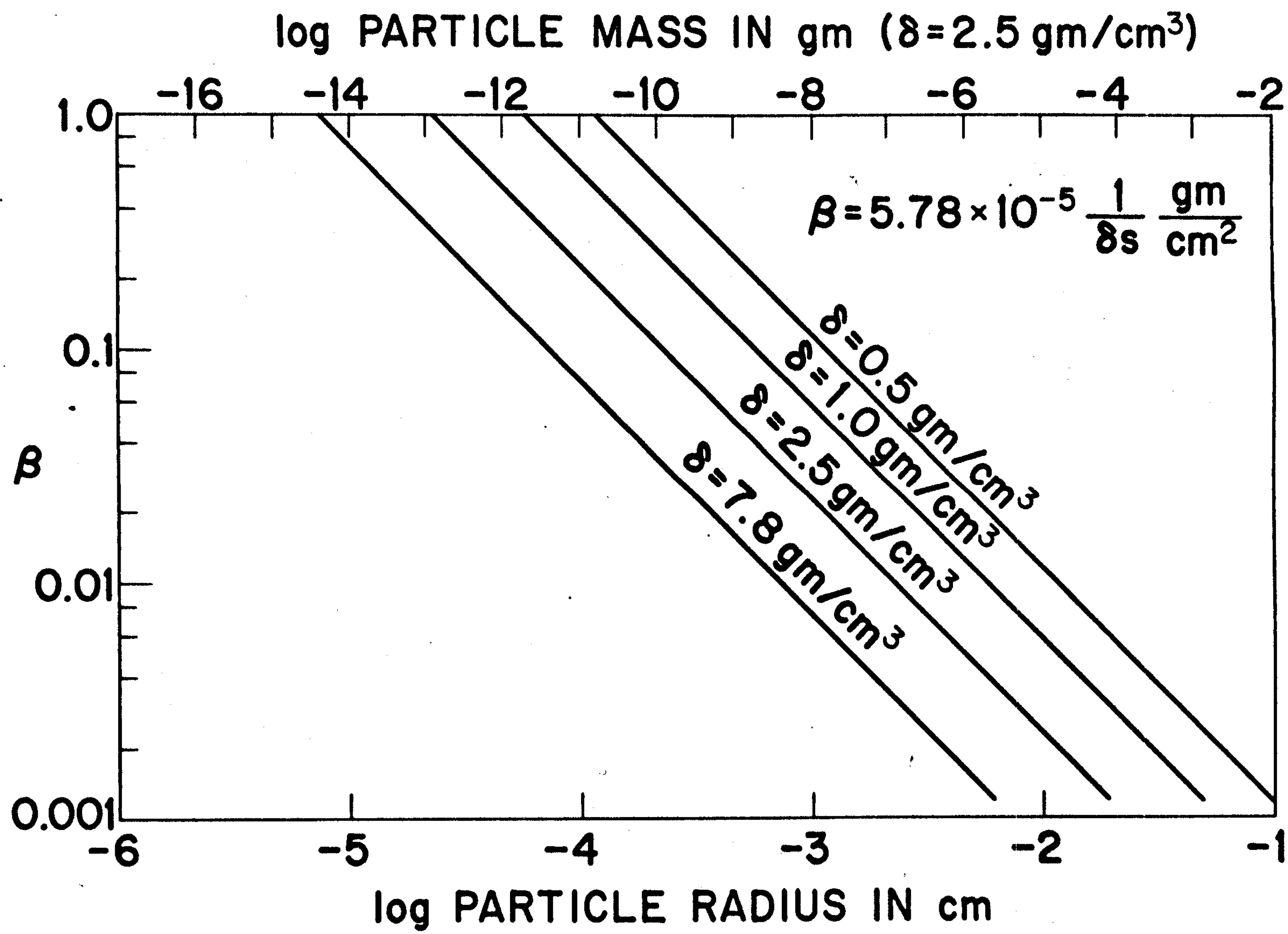
M_v = visual magnitude, M_r = radar magnitude, $M_r \doteq M_v$

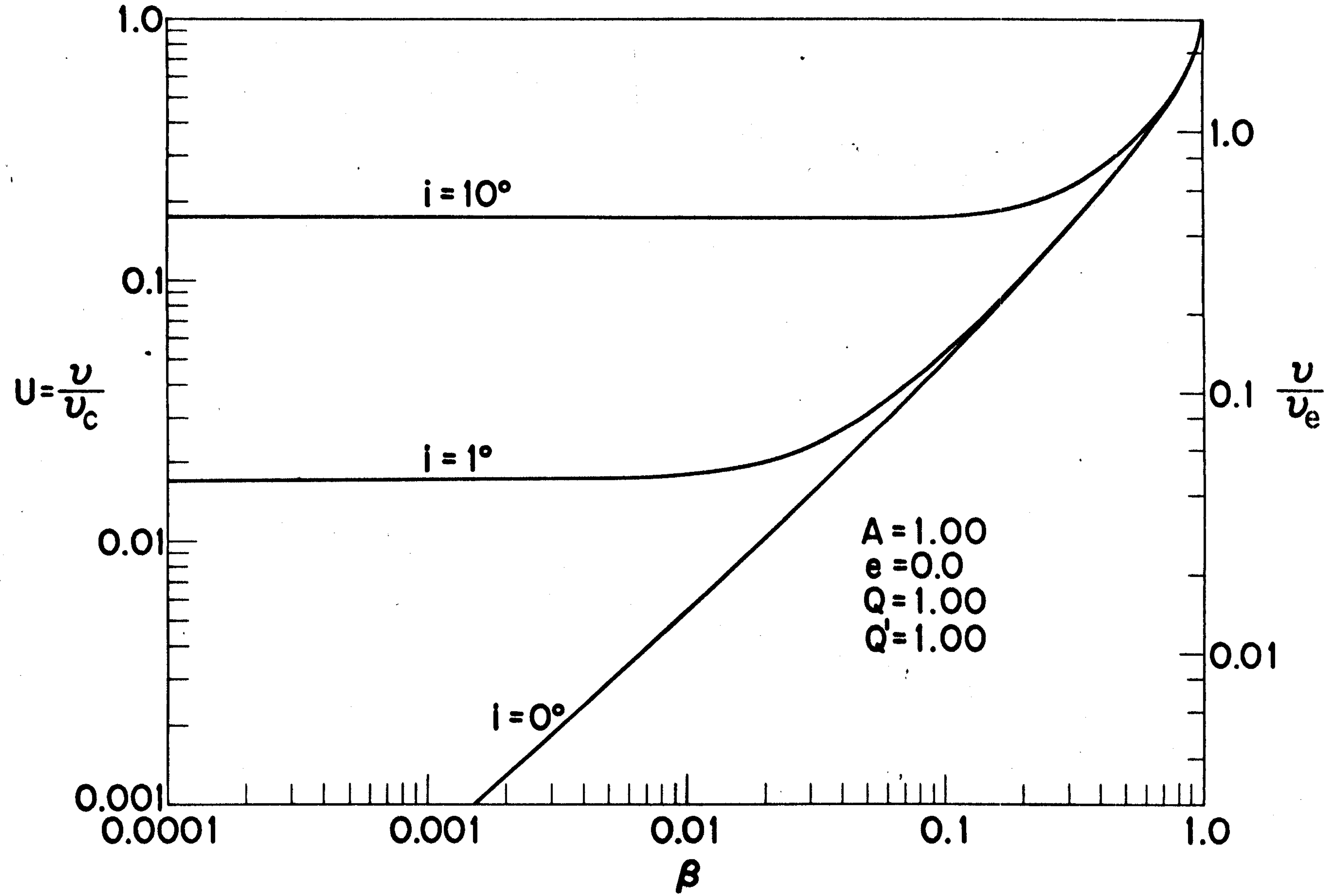
m = particle mass in gm

s = particle radius in cm

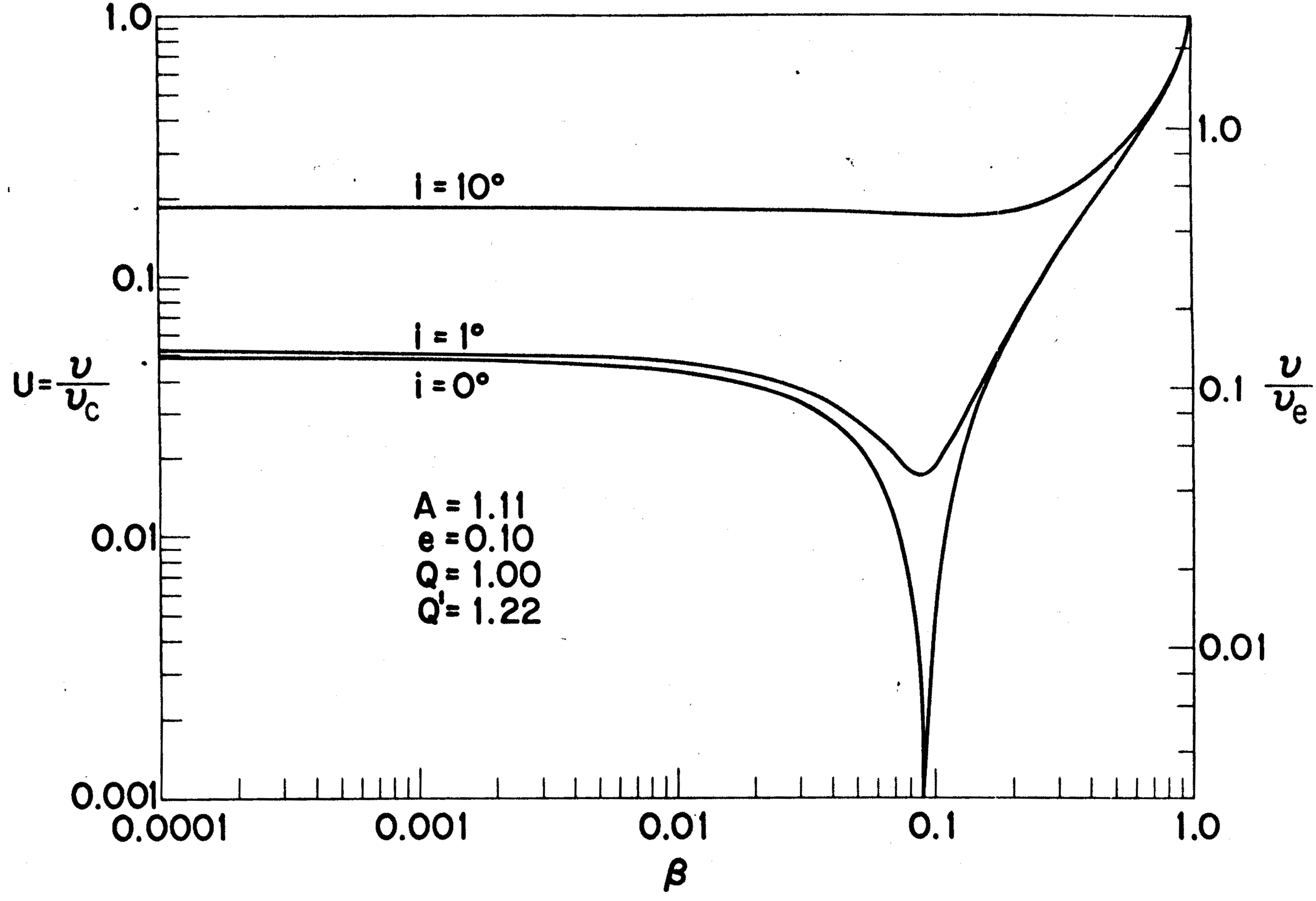


LOG CUMULATIVE INFLUX RATE (particles/m²/sec/2π ster)
LOG CUMULATIVE ACCRETION RATE (particles/earth/day)
McCRACKEN, et al. (1961)
MILLMAN & BURLAND (1956)
INGHAM (1961)
ELSASSER (1954)
KAISER (1961)
WATSON (1956)
HAWKINS & UPTON (1958)
HAWKINS (1959)
BROWN (1961)
VISUAL MAGNITUDE
LOG PARTICLE RADIUS (cm)
LOG PARTICLE MASS (gm)

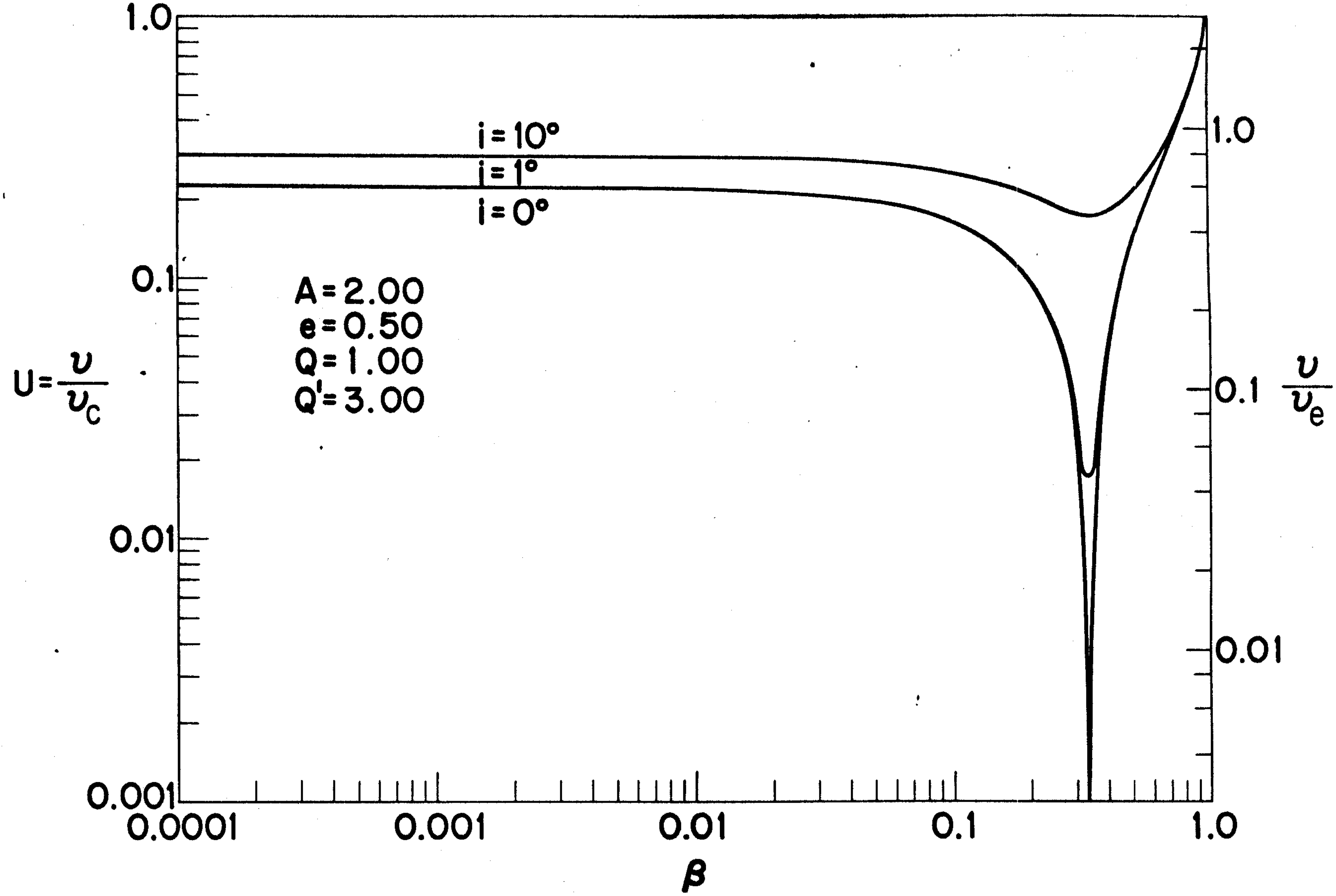

log PARTICLE MASS IN gm (δ=2.5 gm/cm³)
−16
−14
−12
−10
−8
−6
−4
−2
1.0
0.1
0.01
0.001
β
β=5.78×10⁻⁵ 1/δs gm/cm²
δ=0.5 gm/cm³
δ=1.0 gm/cm³
δ=2.5 gm/cm³
δ=7.8 gm/cm³
−6
−5
−4
−3
−2
−1
log PARTICLE RADIUS IN cm



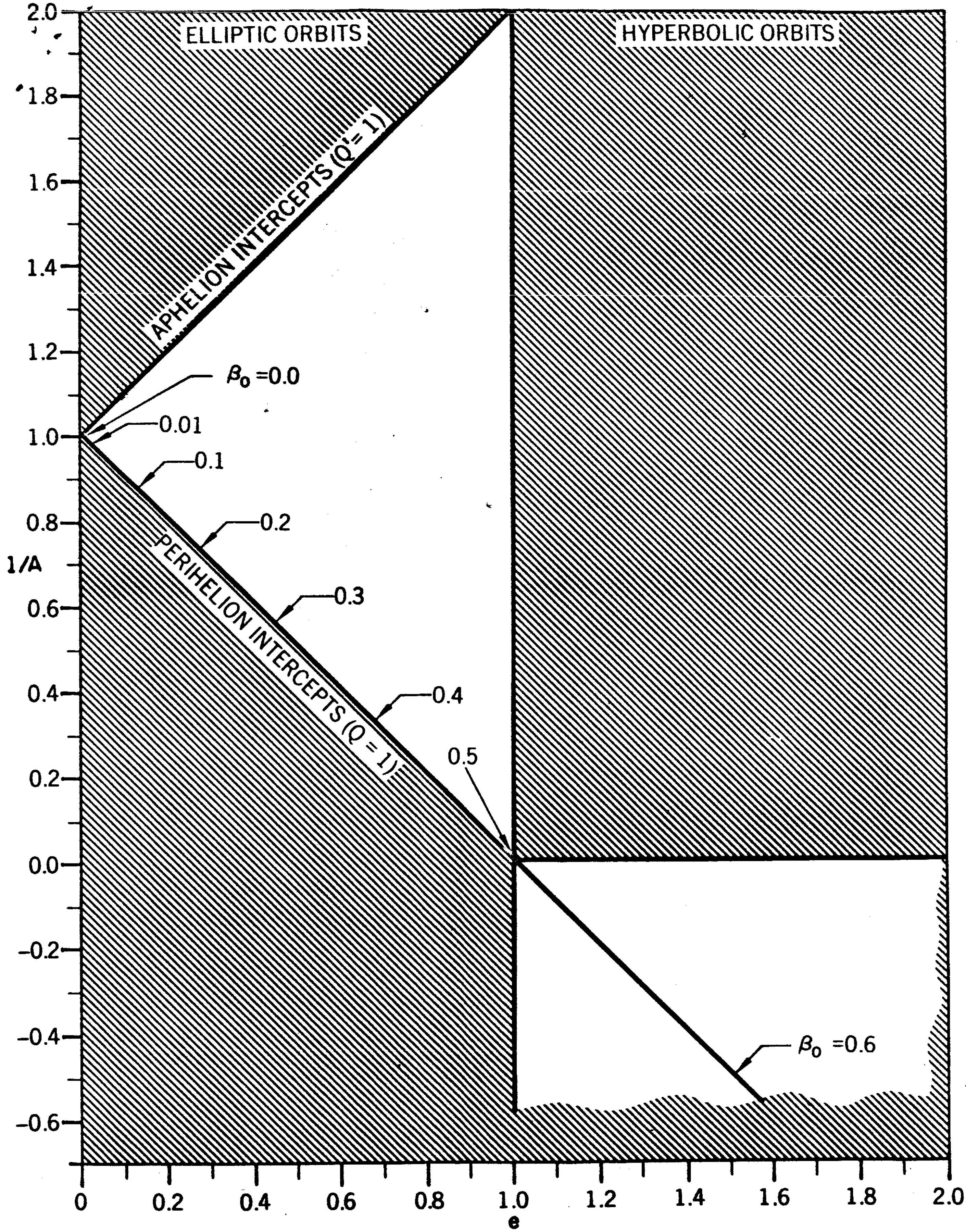
1.0
0.1
0.01
0.001
U = υ/υc
i = 10°
i = 1°
i = 0°
A = 1.00
e = 0.0
Q = 1.00
Q' = 1.00
1.0
0.1
0.01
υ/υe
0.0001
0.001
0.01
0.1
1.0
β



1.0
0.1
0.01
0.001
$U = \frac{v}{v_c}$
0.0001
0.001
0.01
0.1
1.0
β
1.0
0.1
0.01
$\frac{v}{v_e}$
$i = 10°$
$i = 1°$
$i = 0°$
A = 1.11
e = 0.10
Q = 1.00
Q' = 1.22



i = 10°
i = 1°
i = 0°
A = 2.00
e = 0.50
Q = 1.00
Q' = 3.00
U = υ/υc
υ/υe
β
1.0
0.1
0.01
0.001
0.0001



ELLIPTIC ORBITS
HYPERBOLIC ORBITS
APHELION INTERCEPTS (Q' = 1)
PERIHELION INTERCEPTS (Q = 1)
β0 =0.0
0.01
0.1
0.2
0.3
0.4
0.5
β0 =0.6
1/A
e
2.0
1.8
1.6
1.4
1.2
1.0
0.8
0.6
0.4
0.2
0.0
−0.2
−0.4
−0.6
0
0.2
0.4
0.6
0.8
1.0
1.2
1.4
1.6
1.8
2.0